**Department of Computer Science & Engineering**
Indian Institute of Technology Kharagpur

# Class Test – 1

**Soft Computing Applications: CS60108**

*Spring Semester – 2024*

Full Marks: 50 Time:
1 hour

*Answer all questions*

## Problem 1.

1. A membership function of a fuzzy set A is defined as follows

$$\mu_A(x) = \frac{1}{1 + \frac{(x-a)^2}{2k^2}}$$

Where $a$ and $k$ are any two constant values.

Calculate the following.

a) Height
b) Support
c) Crossover point
d) Bandwidth
e) Decide whether $A$ is open or closed.

[5 x 2 = 10 ]

**Solution:**
To calculate the height, support, crossover point, bandwidth, and decide whether the fuzzy set A is open or closed, let's break down the given membership function:

$$\mu_{A(x)} = \frac{1}{1 + \frac{(x-a)^2}{2k^2}}$$

**Height (Maximum Membership Value):** The maximum membership value occurs when x is equal to a. Therefore, the height of the membership function is $\mu_{A(a)} = \frac{1}{1+0} = 1$.

**Support (Range of x for which $\mu_{A(x)} > 0$):** Since the denominator of the function is always positive, the support extends to all real numbers. Therefore, the support is $(-\infty, +\infty)$.

Crossover Point (Point where $\mu_{A(x)} = 0.5$): Let's solve the equation $\mu_{A(x)} = 0.5$ for x:

$0.5 = \frac{1}{1+\frac{(x-a)^2}{2k^2}}$

Or, $1 + \frac{(x-a)^2}{2k^2} = 2$

Or, $\frac{(x-a)^2}{2k^2} = 1$
Or, $(x-a)^2 = 2k^2$

$(x-a) = \pm\sqrt{2k^2}$

$$x = a \pm \sqrt{2}k$$

So, the crossover points are $x = a + \sqrt{2}k$ and $x = a - \sqrt{2}k$

**Bandwidth:** The distance between two distinct cross over points.

$$\left(a + \sqrt{2}k\right) - \left(a - \sqrt{2}k\right) = 2\sqrt{2}k$$

**Open or Closed:** A fuzzy set is closed if its membership function achieves its maximum value at a single point, and open if its maximum value is spread over an interval. In this case, since the maximum membership value occurs at a single point (a), the set A is closed.

**To summarize:**
Height: 1
Support: $(-\infty, +\infty)$
Crossover Points: $= a + \sqrt{2}k$ and $x = a - \sqrt{2}k$
Bandwidth: $2\sqrt{2}k$
Closed set

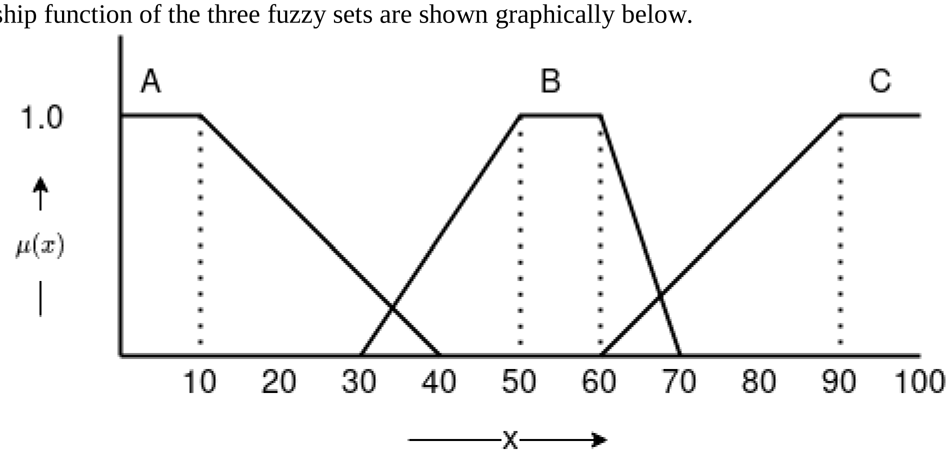
## Problem 2.

Consider a universe of discourse [0 ... 100] in a continuous domain.

Three fuzzy sets are defined over the above universe of discourse:

A: Score is Poor
B: Score is Average
C: Score is Good

The membership function of the three fuzzy sets are shown graphically below.



(a) Express the μ-functions for $\mu_{A(X)}$ , $\mu_{B(X)}$ and $\mu_{C(X)}$ mathematically.

(b) Find the defuzzified value of the **fuzzy set** B using COG method

[2 x 3 + 4 = 10 ]

## Solution:

(a)

$$\mu_{A(x)} = \left\{ \begin{array}{ll} 1 & if\ x \leq 10 \\ \frac{40-x}{40-10} & if\ 10 \leq x \leq 40 \\ 0 & if\ x \geq 40 \end{array} \right\}$$

$$\mu_{B(x)} = \left\{ \begin{array}{ll} 0 & if\ x \leq 30 \\ \frac{x-30}{50-30} & if\ 30 \leq x \leq 50 \\ 1 & if\ 50 \leq x \leq 60 \\ \frac{70-x}{70-60} & if\ 60 \leq x \leq 70 \\ 0 & if\ x \geq 70 \end{array} \right\}$$

$$\mu_{C(x)} = \left\{ \begin{array}{ll} 0 & if\ x \leq 60 \\ \frac{x-60}{90-60} & if\ 60 \leq x \leq 90 \\ 1 & if\ x \geq 90 \end{array} \right\}$$

(b)

The defuzzified value of set B, with COG method is:

$$z^* = \frac{\int \mu_{B(Z)} z dz}{\int \mu_{B(Z)} dz}$$

$$z^* = \frac{\int_{30}^{50} \frac{(z-30)}{20} zdz + \int_{50}^{60} 1.zdz + \int_{60}^{70} \frac{(70-z)}{10} zdz}{\int_{30}^{50} \frac{(z-30)}{20} dz + \int_{50}^{60} 1.dz + \int_{60}^{70} \frac{(70-z)}{10} dz}$$

$$= \mathbf{\frac{1300}{25}} = \mathbf{52}$$

## Problem 3.

Following are the three fuzzy sets known.

Score is high: { (50,0.3), (60,0.4), (70,0.5), (80,0.8), (90,0.9), (100,1.0)}

IQ is high: { (2,0.1), (3,0.2), (4,0.6), (5,0.8), (6,0.9)}

IQ is low: { (1,0.9), (2,0.8), (3,0.7), (4,0.2), (5,0.1), (6,0)}

Given the above fuzzy set, how you can deduce **Score is Low.**

Assume that there is a rule that if score is High Then IQ is High.

[5 + 10 = 15 ]

## Solution:

Assume, x is score and y is IQ. Also, A and B implies x and y are high, respectively.

A = {(50,0.3), (60,0.4), (70,0.5), (80,0.8), (90,0.9), (100,1)}

B = {(1,0), (2,0.1), (3,0.2), (4,0.6), (5,0.8), (6,0.9)}

$\underline{B}$ = {(1,0.9), (2,0.8), (3,0.7), (4,0.2), (5,0.1), (6,0)}

if score is High Then IQ is High = $A \rightarrow B = (A\times B) \cup \underline{A} \times Y$:

A×B:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0.1 | 0.2 | 0.3 | 0.3 | 0.3 |
| 0 | 0.1 | 0.2 | 0.4 | 0.4 | 0.4 |
| 0 | 0.1 | 0.2 | 0.5 | 0.5 | 0.5 |
| 0 | 0.1 | 0.2 | 0.6 | 0.8 | 0.8 |
| 0 | 0.1 | 0.2 | 0.6 | 0.8 | 0.9 |
| 0 | 0.1 | 0.2 | 0.6 | 0.8 | 0.9 |

$\underline{A} \times Y$:

| | | | | | |
|---|---|---|---|---|---|
| 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 0 | 0 | 0 | 0 | 0 | 0 |

R(x,y):

| | | | | | |
|---|---|---|---|---|---|
| 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 0.2 | 0.2 | 0.2 | 0.6 | 0.8 | 0.8 |
| 0.1 | 0.1 | 0.2 | 0.6 | 0.8 | 0.9 |
| 0 | 0.1 | 0.2 | 0.6 | 0.8 | 0.9 |

$\underline{B} = [0.9 \;\; 0.8 \;\; 0.7 \;\; 0.2 \;\; 0.1 \;\; 0]$

$\underline{B} \circ R(x,y): [0.7 \;\; 0.6 \;\; 0.5 \;\; 0.2 \;\; 0.2 \;\; 0.2]$

$\underline{A}$ = {(50,0.7), (60,0.7), (70,0.7), (80,0.7), (90,0.7), (100,0.7)}

## Problem 4.

Bipolar sigmoid transfer function for a given value of α is given below

$$\Phi(I) = \frac{e^{-\alpha I} - e^{+\alpha I}}{e^{-\alpha I} + e^{+\alpha I}}$$

(a) Find the limiting value of $\Phi(I)$

(b) Draw the function graphically and show how it looks for different values of α.

(c) Prove that

$$\frac{\partial \Phi(I)}{\partial I} = \alpha(1 - \Phi(I))(1 + \Phi(I))$$

[5+5+5=15]

## Solution:

4 (a).

$$\phi(I) = \frac{e^{-\alpha I} - e^{\alpha I}}{e^{-\alpha I} + e^{\alpha I}}$$

- $$\lim_{I\to\infty} \frac{e^{-\alpha I} - e^{\alpha I}}{e^{-\alpha I} + e^{\alpha I}} = \lim_{I\to\infty} \frac{1 - \frac{e^{\alpha I}}{e^{-\alpha I}}}{1 + \frac{e^{\alpha I}}{e^{-\alpha I}}}$$
$$= \lim_{I\to\infty} \frac{1 - e^{2\alpha I}}{1 + e^{2\alpha I}}$$

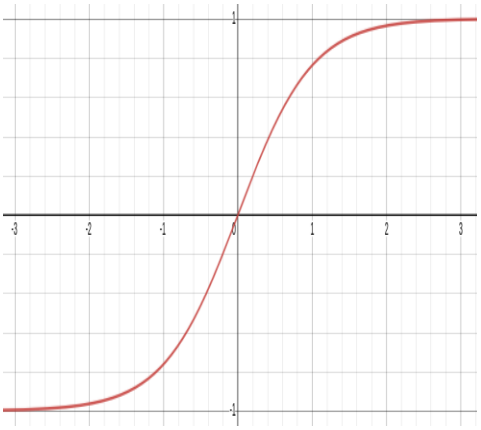
Using L'Hopital rule:

$$\lim_{I\to\infty} \frac{1 - e^{2\alpha I}}{1 + e^{2\alpha I}} = \lim_{I\to\infty} \frac{0 - 2e^{2\alpha I}.I}{0 + 2e^{2\alpha I}.I} = -1$$

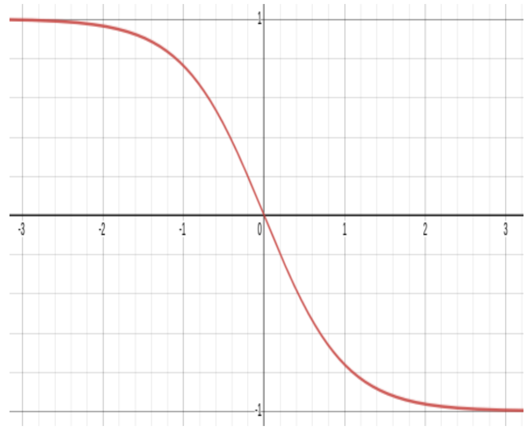
- $$\lim_{I\to-\infty} \frac{e^{-\alpha I} - e^{\alpha I}}{e^{-\alpha I} + e^{\alpha I}} = \lim_{I\to-\infty} \frac{1 - \frac{e^{\alpha I}}{e^{-\alpha I}}}{1 + \frac{e^{\alpha I}}{e^{-\alpha I}}}$$
$$= \lim_{I\to-\infty} \frac{1 - e^{2\alpha I}}{1 + e^{2\alpha I}} = \frac{1 - 0}{1 + 0} = 1$$

Therefore, the range is [-1,1]

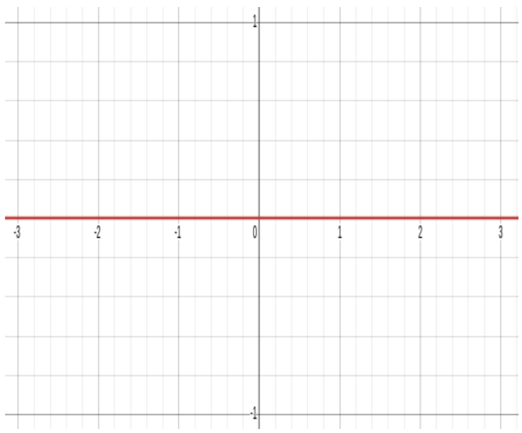
4 (b).



Case $\alpha < 0$



Case $\alpha > 0$



Case $\alpha = 0$

4 (c)

To probe $\frac{\partial \Phi(I)}{\partial I} = \alpha(1 - \Phi(I))(1 + \Phi(I))$

. $\frac{\partial \phi(I)}{\partial I} = \frac{\partial}{\partial I}\left[\frac{e^{-\alpha I} - e^{\alpha I}}{e^{-\alpha I} + e^{\alpha I}}\right]$

$= \frac{(-\alpha e^{-\alpha I} - \alpha e^{\alpha I})(e^{-\alpha I} + e^{\alpha I}) - (e^{-\alpha I} - e^{\alpha I})(\alpha e^{-\alpha I} + \alpha e^{\alpha I})}{(e^{-\alpha I} + e^{\alpha I})^2}$

$= \frac{(-\alpha)(e^{-\alpha I} + e^{\alpha I})^2 - (-\alpha)(e^{-\alpha I} - \alpha e^{\alpha I})^2}{(e^{-\alpha I} + e^{\alpha I})^2}$

$= (-\alpha)\left[\frac{(e^{-\alpha I} + e^{\alpha I})^2 - (e^{-\alpha I} - e^{\alpha I})^2}{(e^{-\alpha I} + e^{\alpha I})^2}\right]$

$= (-\alpha)\left[\frac{(e^{-\alpha I} + e^{\alpha I})^2}{(e^{-\alpha I} + e^{\alpha I})^2} - \frac{(e^{-\alpha I} - e^{\alpha I})^2}{(e^{-\alpha I} + e^{\alpha I})^2}\right]$

$= (-\alpha)[1 - (\phi(I))^2]$

$= (-\alpha)[(1 + \phi(I))(1 - \phi(I))]$